Foreword

Artificial Intelligence (AI) has recently made unexpected, sometimes far-reaching, progress in solving open problems in some areas of Applied Mathematics, prompting some applied mathematicians to wonder whether this was the beginning of the end of Applied Mathematics as we know it.

Actually, it is the opposite that is true, in the sense that AI has evolved as a source of numerous, new, and quite challenging, problems in Applied Mathematics, thus contributing to a revival of our science! This aspect of AI should definitely convince our colleagues and students that they should invest their time in trying to solve such problems.

At the highest mathematical level, this book is a perfect illustration of this recent trend, and as such one of the very first of its kind.

Its main focus is to explore in depth the connections between Machine Learning and Control theory, particularly by way of Stochastic Control Theory. Modern Control theory, which began in the late fifties with the space race as its main motivation, provides the mathematical tools for making decisions for dynamical systems subjected to uncertainties; indeed the applications of Control Theory to Engineering and Operations Research have been overwhelming during the past decades.

The last decade saw the rapid development of Machine Learning. Among its significant breakthroughs, one can mention Deep Learning, the propagation of the gradient algorithm, or the concept of adjoint state familiar to control theorists; incidentally, what is now called Machine Learning is basically identical to what is called System Identification in Control Theory. Neural networks constitute the backbone of Machine Learning. They are characterized by the number of neurons and the number of layers. When these numbers are very large, adapting a neural network to data becomes a problem of control for dynamical systems governed by differential equations or by partial differential equations. These are the topics that this book thoroughly analyze. Among its highlights: The use of kernels in machine learning was very popular before the introduction of deep learning. They were introduced in statistics and approximation theory and used largely in machine learning. This beautiful theory is very well analyzed in the book.

Although problems in Supervised Learning and Deep Learning are commonly considered as deterministic, they are fundamentally stochastic. The reason comes from the origin of the data, which are basically samples arising from some distribution, the empirical one at the very least. An important issue is to know more about this distribution, for instance so as to be able to generate more data by simulation. This fundamental objective of Generative AI, a new branch of AI growing rapidly, is analyzed in the book.

The book also explains how Bayesian Learning, so useful in many situations, has the crucial advantage of simultaneously performing learning and decision making.

It shows how stochastic approximation and stochastic gradient, both very fast developing research areas for many years, are now rejuvenated by Machine Learning.

The book also contains results about identification of dynamical systems in the spirit of machine learning ideas and applications to inventory learning and control. It also shows that Reinforcement

Learning is very close to the Markov Decision Processes.

This brief description shows that the book admirably fulfills its objectives, by providing complete and rigorous mathematical justification of many notions and methods, some quite recent, connecting control theory and machine learning.

The authors are to be commended for having written such a book, which will undoubtedly render a signal service to both the Applied Mathematics community and the AI community.

I sincerely wish that this book meets with all the success that it so obviously deserves!

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