Math 421 - Homework 9

Reading assignment: Chapters 10-11 of the Spivak textbook.

Written HW Assignment: Please write your solution to each problem on a separate page, with your name and the full problem statement at the top of the page. Your solutions to all problems should be written in complete sentences, with proper grammatical structure. Typed solutions would be added one extra point in this homework.

1. Determine all local maximum and local minimum points for the following functions on R. (Assume that the derivative of sine is cosine and the usual properties of the cosine functions.)
   a. $f(x) = x(x^2 - 1)(x^2 - 4)$
   b. $g(x) = sinx - x$
   c. $h(x) = sin(x^2)$

2. Rolle's Theorem requires three assumptions on a function f:
   a. *The function f is defined on an interval I with endpoints.*
   b. *The function f is continuous on the interval I.*
   c. *The function f is differentiable on the interval I except at the endpoints.*
   Show by three counterexamples that you cannot drop any of these three italicized assumptions.

3. Assume that f is continuous on $[a, b]$ and differentiable on (a, b). Assume also that $f'(x) \neq 0$ on (a, b) and $f(a)$ and $f(b)$ have different signs. Show that the equation $f(x) = 0$ has a unique solution in (a, b).