

## Math 421 - Homework 8

**Reading assignment:** Chapters 10-11 of the Spivak textbook.

**Written HW Assignment:** Please write your solution to each problem on a separate page, with your name and the full problem statement at the top of the page. Your solutions to all problems should be written in complete sentences, with proper grammatical structure. Typed solutions would be added one extra point in this homework.

1. Find the derivatives of the following functions (assuming the usual derivative rules for sin and cos):

a.

$$f(x) = \sqrt{\sin(x^2) + \cos^2(x)}$$

b.

$$g(x) = \frac{(\sin x - \cos^2 x)^2}{\sqrt{x^3 - x}}.$$

2. Let

$$f(x) = \begin{cases} x^2 & \text{if } x \text{ is rational} \\ 0 & \text{otherwise} \end{cases}$$

(a) Is  $f$  differentiable at 0? Justify.

(b) Is  $f$  differentiable at  $x \neq 0$ ? Justify.

3. Suppose that  $f'(0)$  exists and  $f(x+y) = f(x)f(y)$  for all  $x$  and  $y$ . Prove that  $f'(x)$  exists for all  $x$ . Find  $f'(x)$  for all  $x$ .
4. Use the Product Rule and that  $\frac{d}{dx}x^n = nx^{n-1}$ ,  $n \in \mathbb{Z}^+$  to show that  $f : \mathbb{R}^+ \rightarrow \mathbb{R}$  given by  $f(x) = x^{n+1/2}$ ,  $n \in \mathbb{Z}^+$  is differentiable, and find  $f'(x)$ .