

## Math 421 - Homework 6

**Reading assignment:** Chapters 7 and 8 of the Spivak textbook.

**Written HW Assignment:** Please write your solution to each problem on a separate page, with your name and the full problem statement at the top of the page. Your solutions to all problems should be written in complete sentences, with proper grammatical structure. Typed solutions would be added one extra point in this homework.

1. Let  $\lim_{x \rightarrow x_0} f(x) = l$  and  $\lim_{x \rightarrow x_0} g(x) = m$ . Show that

$$(a) \lim_{x \rightarrow x_0} \max\{f(x), g(x)\} = \max\{l, m\};$$

$$(b) \lim_{x \rightarrow x_0} \min\{f(x), g(x)\} = \min\{l, m\}.$$

2. Show that  $x^2 + 2 = e^x$  has at least one solution in  $\mathbb{R}$ .

3. Let  $f : (-1, 1) \rightarrow \mathbb{R}$  be continuous at  $x = 0$  and suppose that  $f(x) = f(x^2)$  for all  $x \in (-1, 1)$ . Show that  $f(x) = f(0)$  for all  $x \in (-1, 1)$ .

4. Let  $f : [a, b] \rightarrow \mathbb{R}$  be continuous and suppose that there is some  $c \in (a, b)$  such that  $f(c) \neq 0$ . Show that there exists some  $\delta > 0$  such that  $f(x) \neq 0$  for every  $x \in (c - \delta, c + \delta)$ .

5. Show that there is no continuous function  $f : \mathbb{R} \rightarrow \mathbb{R}$  with  $f(\mathbb{R}) = \mathbb{Q}$ .