

Math 421 - Homework 5

Reading assignment: Chapters 6 and 7 of the Spivak textbook.

Written HW Assignment: Please write your solution to each problem on a separate page, with your name and the full problem statement at the top of the page. Your solutions to all problems should be written in complete sentences, with proper grammatical structure. Typed solutions would be added one extra point in this homework.

1. For what values of a, b and c is $f(x) = ax + b$, if $x > 1$, $f(x) = cx^2$, if $x \leq 1$ continuous at 1?
2. Give an example of a function $f : \mathbb{R} \rightarrow \mathbb{R}$, that is discontinuous at all $x \in \mathbb{R}$, such that $|f(x)|$ is continuous for all x .
3. Suppose that f and g are two functions continuous on \mathbb{R} such that $|f| = |g|$ and such that $f(x) \neq 0$ and $g(x) \neq 0$ for all $x \in \mathbb{R}$. Show that $f = g$ or $f = -g$.
4. Let $c > 0$ and $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfy

$$|f(x) - f(y)| \leq c|x - y|$$

for all $x, y \in \mathbb{R}$.

Show that f is continuous.

Deduce that cosine is a continuous function.

5. Prove that every continuous function f can be written $f = g - h$, where g and h are nonnegative and continuous.