

Math 421 - Homework 4

Reading assignment: Chapters 5 and 6 of the Spivak textbook.

Written HW Assignment: Please write your solution to each problem on a separate page, with your name and the full problem statement at the top of the page. Your solutions to all problems should be written in complete sentences, with proper grammatical structure. Typed solutions would be added one extra point in this homework.

1. Suppose $f(x) > 0$ for all x , and also that $\lim_{x \rightarrow a} f(x)$ exists.
 - a) Show that $\lim_{x \rightarrow a} f(x) \geq 0$.
 - b) Give an example where $\lim_{x \rightarrow a} f(x) = 0$.
2. Prove that $\lim_{x \rightarrow 2} \frac{3}{x+1} = 1$ directly by using the definition of the limit.
3. Prove that $\lim_{x \rightarrow 2} \frac{1}{x+1} = \frac{1}{3}$ directly from the definition, and not by using a theorem.
4. Give an example to show that the following definition of $\lim_{x \rightarrow a} f(x) = L$ is *NOT* correct:
For all $\epsilon > 0$ there is a $\delta > 0$ such that if $|f(x) - L| < \epsilon$, then $0 < |x - a| < \delta$.
5. Prove or give a counter example: if $|f|$ is continuous at a , then f is continuous at a
6. Prove or give a counter example: if f is continuous at a , then $|f|$ is continuous at a