

## Math 421 - Homework 3

**Reading assignment:** Chapters 3 and 4 of the Spivak textbook.

**Written HW Assignment:** Please write your solution to each problem on a separate page, with your name and the full problem statement at the top of the page. Your solutions to all problems should be written in complete sentences, with proper grammatical structure. Typed solutions would be added one extra point in this second homework.

1. Call a function  $f : \mathbb{R} \rightarrow \mathbb{R}$  even if for all  $x \in \mathbb{R}$ ,  $f(x) = f(-x)$ ; and odd if for all  $x \in \mathbb{R}$ ,  $f(x) = -f(-x)$ . Show:
  - a. The sum of two even functions is even; and the sum of two odd functions is odd.
  - b. The product of two even functions is even; and the product of two odd functions is even.
  - c. The product of an even function and an odd function is odd.
  - d. Must the sum of an even function and an odd function be even? Odd?
2. Determine whether the following functions are even, odd, or neither:
  - a.  $f(x) = x^5$ .
  - b.  $f(x) = x^5 + x^{10} + 500x^{35}$ .
  - c.  $f(x) = x^2 + x^{13}$ .
  - d.  $f(x) = \cos(\sin(x^{100}))$ .
3. Call a function  $f : \mathbb{R} \rightarrow \mathbb{R}$  periodic (with period  $A$ ) if for all  $x \in \mathbb{R}$ ,  $f(x) = f(x + A)$ .
  - a. Show that the sum of periodic functions with period  $A$  has period  $A$ .
  - b. Show that the sum of periodic functions, one with period  $2A$  and the other with period  $3A$ , has period  $6A$ .
4. Determine whether the following functions are periodic, and if so, what the (smallest) period is:
  - a.  $f(x) = \sin(x)$ .
  - b.  $f(x) = \cos(\sin(x^{100}))$ .
  - c.  $f(x) = \cos(x) + \sin(2x)$ .