# Math 421 - Homework 3

**Reading assignment:** Chapters 2 and 3 of the Spivak textbook.

**Written HW Assignment:** Please write your solution to each problem on a separate page, with your name and the full problem statement at the top of the page. Your solutions to all problems should be written in complete sentences, with proper grammatical structure. Typed solutions would be added one extra point in this second homework.

1. Prove by induction: If $h > -1$ then $(1+h)^n \geq 1 + nh$.

2. Prove the following modification of the Principle of Mathematical Induction: If $A \subset \mathbb{N}$ contains a number $n_0 \in \mathbb{N}$ and for all $k \in \mathbb{Z}$, $k \in A$ implies $k+1 \in A$, then $A$ contains all natural numbers $\geq n_0$.

3. In Mathland the currency consists of 3 and 4 penny coins. Suppose you want to buy an item that costs 9 pennies, then you would be paying with three 3 penny coins.
   Show that if you have an unlimited number of 3 and 4 penny coins you can pay any price greater than or equal to six pennies, without receiving change.
   Hint: use complete induction.

4. Show that $\sqrt{15}$ is irrational.