

Math 421 - Homework 2

Reading assignment: Chapters 1 and 2 of the Spivak textbook.

Written HW Assignment: Please write your solution to each problem on a separate page, with your name and the full problem statement at the top of the page. Your solutions to all problems should be written in complete sentences, with proper grammatical structure. Typed solutions would be added one extra point in this second homework.

1. Prove that for $a \in \mathbb{R}$

a. For $0 < a < 2$:

$$a^2 < 2a.$$

b. For $-3 < a < 0$:

$$a^2 < 9.$$

c.

$$a^2 + 2a + 1 \geq 0.$$

2. Find all numbers $x \in \mathbb{R}$ for which

a. $|x + 2| > 3$.

b. $|x + 5| > 9$.

c. $x^2 + 4x + 4 > 1$.

3. Prove that for $x, y \in \mathbb{R}$

a.

$$x^2 + 2xy + y^2 \geq 0.$$

b.

$$x^4 + 3x^2y^2 + y^4 \geq 0.$$

4. Prove by mathematical induction that

$$1 + 3 + 5 + \cdots + (2n + 1) = (n + 1)^2.$$

5. For two sets A and B show that the following statements are equivalent:

a) $A \subset B$

b) $A \cup B = B$

c) $A \cap B = A$